

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \text{ for-re ismeret-}$$

hét feladat

$$(1) \lim_{x \rightarrow 0} \frac{x}{\sin x} = \lim_{x \rightarrow 0} \frac{1}{\frac{\sin x}{x}} = \frac{1}{1} = 1$$

$$(2) \lim_{x \rightarrow 0} \frac{\sin^2 x}{x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \sin x = 0$$

$$(3) \lim_{x \rightarrow 0} \frac{\sin 2x}{x} = \lim_{x \rightarrow 0} 2 \frac{\sin 2x}{2x} = 2$$

$$(4) \lim_{x \rightarrow 0} \frac{1}{\cos x} = \lim_{x \rightarrow 0} \frac{\sin x}{\cos x} = \lim_{x \rightarrow 0} \frac{1}{\cos x} \cdot \frac{\sin x}{x} = 1$$

$$(5) \lim_{x \rightarrow 0} \frac{\sin^2 \frac{x}{2}}{x^2} = \lim_{x \rightarrow 0} \left(\frac{\sin \frac{x}{2}}{\frac{x}{2}} \right)^2 \cdot \frac{1}{4} = \frac{1}{4}$$

$$(6) \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{x}{2}}{x} = \lim_{x \rightarrow 0} \frac{\sin^2 \frac{x}{2}}{\frac{x}{2}} =$$

$$= \lim_{x \rightarrow 0} \frac{\sin \frac{x}{2}}{\frac{x}{2}} \cdot \sin \frac{x}{2} = 1 \cdot 0 = 0$$