

A $\lim_{x \rightarrow +\infty} \left(1 + \frac{1}{x}\right)^x = e$ -re ismeret -
hazó feladat

① $\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} =$

$x = \frac{1}{t} \Rightarrow$ ha $x \rightarrow 0$, akkor $t \rightarrow +\infty$

$\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = \lim_{x \rightarrow +\infty} \left(1 + \frac{1}{t}\right)^t = e$

$\Rightarrow \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e$

② $\lim_{x \rightarrow -\infty} \left(1 + \frac{1}{x}\right)^x$

$x = -(z+1)$

ha $x \rightarrow -\infty$, $z \rightarrow +\infty$

$\lim_{x \rightarrow -\infty} \left(1 + \frac{1}{x}\right)^x = \lim_{z \rightarrow +\infty} \left(1 - \frac{1}{z+1}\right)^{-(z+1)} =$

$= \lim_{z \rightarrow +\infty} \left(\frac{z+1-1}{z+1}\right)^{-(z+1)} = \lim_{z \rightarrow +\infty} \left(\frac{z}{z+1}\right)^{-(z+1)} =$

$= \lim_{z \rightarrow +\infty} \left(\frac{z+1}{z}\right)^{z+1} = \lim_{z \rightarrow +\infty} \underbrace{\left(1 + \frac{1}{z}\right)^z}_e \cdot \underbrace{\left(1 + \frac{1}{z}\right)}_1 = e$

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